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HS-5000: Stochastic Method Comparison and Reliability-Based Design Optimization of the Arm Model

Continuing from tutorial HS-4000: Optimization Method Comparison: Arm Model Shape Optimization, you will perform Stochastic studies and a Reliability-Based Design Optimization (RBDO) using the same fitting function. You will run a Stochastic study around the nominal point.

Before running this tutorial, complete tutorial Tutorial HS-4000: Optimization Method Comparison: Arm Model Shape Optimization. You can also import the archive file <code>HS-4000.hstx</code>, available in <hst.zip>/HS-5000/.

In the Stochastic study, you will be using a Hammersley distribution successively with 100, 300, and 1000 runs in order to compare the convergence of statistical results. You defined all six input variables as random variables following a normal distribution.

Step 1: Run a Stochastic Study

In this step, you will check the robustness of the optimal solution found with GRSM.

- 1. In the **Explorer**, right-click and select **Add** from the context menu.
- 2. In the Add HyperStudy dialog, select Stochastic and click OK.
- 3. Go to the **Select Input Variables** step.
- 4. In the **Active** column, clear the *radius_1*, *radius_2* and *radius_3* check boxes.

	Active	Label	Varname	Low
1	V	length_1	m_1_length_1	-0.500
2	V	length_2	m_1_length_2	0.000
3	V	length_3	m_1_length_3	-1.000
4	V	length_4	m_1_length_4	-1.000
5	V	length_5	m_1_length_5	-1.000
6		radius_1	m_1_radius_1	-2.000
7		radius_2	m_1_radius_2	-0.500
8		radius_3	m_1_radius_3	-0.500
9	V	height	m_1_height	-1.000

5. In the **Nominal** column, copy the parameter values at the optimal design.

For Stochastic studies, you must provide data about the standard variation σ (or variance σ^2) of parameters in order to take into account uncertainties. By default, σ^2 is computed in HyperStudy using the range rule $\sigma^2 = ((Upper Bound-Lower Bound)/4)^2$ which is a function of the input variable's bounds. If you do not have reliable data about the standard deviation, the default σ^2 can be modified by changing the upper and lower bounds of the parameters.

 a. Go to the GRSM, 6 IV, Exact Solver Optimization, Evaluate step, Iteration History tab, and copy the length_1, length_2, length_3, length_4, and length_5 parameter values.



	"]+ length_1	"]+ length_2	"]+ length_3	"]+ length_4	"]+ length_5	¶+ h
1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.00000
2	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.00000
3	-0.4992779	0.0016672	-0.9912966	0.0498490	0.7815539	0.99908
4	-0.4291008	5.86e-04	-0.9886176	0.2059647	0.4663968	0.99951
5	-0.2325116	1.34e-04	-0.9993315	0.3377470	0.2309717	0.99995
6	-0.2325116	1.34e-04	-0.9993315	0.3377470	0.2309717	0.99995
7	-0.4764394	0.0000000	-1.000000	0.4958683	0.0347269	1.00000
8	-0.4764394	0.0000000	- <mark>1</mark>	down	9	1.00000
9	-0.4764394	0.0000000	-1 👌 Sort	up)	1.00000
10	-0.4764394	0.0000000	-1)	1.00000
11	-0.4764394	0.0000000	-1 Fit C	Columns	9	1.00000
12	-0.4764394	0.0000000	-1 Colu	umns) ► D	1.00000
13	-0.4764394	0.0000000	-1)	1.00000
14	-0.4764394	0.0000000	-1 🖺 Cop	y Ctrl+	C P	1.00000
15	-0.4764394	0.0000000	-1 💽 🛛 Cop	y + labels	9	1.00000
10	0.1761004	0.000000	4			1 00000

b. Go to the Stochastic study, Select Input Variables step, and select the Nominal fields for length_1, length_2, length_3, length_4, and length_5. Right-click on the selected fields, and select Paste transpose fro the context menu.

	Active	Label	Varname	Lower Bound	Nomina	ıl	Upper Bound	Comment	
1	\checkmark	length_1	m_1_length_1 📶	-0.5000000	-0.4764394	0	Add Input Vari	able	
2		length_2	m_1_length_2	0.0000000	0.0000000	Ŭ	nuu nput run	0010	
3	\checkmark	length_3	m_1_length_3 📶	-1.0000000	-1.0000000		Remove Input		
4	\checkmark	length_4	m_1_length_4 📶	-1.0000000	0.4958683	81	Sort down		
5	\checkmark	length_5	m_1_length_5 👖	-1.0000000	0.0347269				
6		radius_1	m_1_radius_1	-2.0000000	0.0000000	ğŢ	Sort up		
7		radius_2	m_1_radius_2	-0.5000000	0.0000000		Fit Columns		
8		radius_3	m_1_radius_3	-0.5000000	0.0000000				
9	\checkmark	height	m_1_height	-1.0000000	0.0000000		Columns		
							Сору	Ctrl+C	
							Copy + labels		
						Ê	Paste	Ctrl+V	
						1	Paste transpos	e 🔓	
						V	Filter		•

- c. For height, change the Nominal value to 1.0.
- 6. Change the lower and upper bounds for every active input variable to match the dimensions' tolerances. For this tutorial, assume that the tolerances are within 0.05 mm.



- a. For all active input variables, click •••• in the **Nominal** field.
- b. In the pop-up window, Value field, enter 0.05 and click +/-.

Lower Bou	nd	Nominal	Upper Bound
-0.5264394		-0.4764394	-0.4264394
Set Range			
Percent:			+/-
Value:	.05		+/-
		ОК	Cancel Apply

c. Click **OK** to accept the changes and close the pop-up window.

The Lower and Upper bounds for all active input variables should replicate the image below.

	Active	Label	Varname	Lower Bound	Nominal	Upper Bound	Comment
1	1	length_1	m_1_length_1	-0.5264394	-0.4764394	-0.4264394	
2	1	length_2	m_1_length_2	-0.0500000	0.0000000	0.0500000	
3	1	length_3	m_1_length_3	-1.0500000	-1.0000000	-0.9500000	
4	1	length_4	m_1_length_4	0.4458683	0.4958683	0.5458683	
5	1	length_5	m_1_length_5	-0.0152731	0.0347269	0.0847269	
6		radius_1	m_1_radius_1	-2.0000000	0.0000000	2.0000000	
7		radius_2	m_1_radius_2	-0.5000000	0.0000000	1.0000000	
8		radius_3	m_1_radius_3	-0.5000000	0.0000000	1.0000000	
9	1	height	m_1_height	0.9500000	1.0000000	1.0500000	

7. Click the *Distributions* tab, and verify that **Distribution** is set to **Normal Variance** for all active input variables.

Look at the columns A and B. Column A displays the nominal parameter values, and column B displays the variance σ^2 which is computed using the standard deviation σ of a parameter around its mean.

Variance is computed as follows: $\sigma^2 = ((Upper Bound-Lower Bound)/4)^2$. For instance, the variance for the input variable length_1 is $\sigma^2 = ((-0.45+0.55)/4)^2 = 6.25e-4$.



	Active	Label	Varname		Distribution Role	Distribution	А	
1	1	length_1	m_1_length_1	հ	Random Parameter	Normal Variance 🔺	-0.4764394	6.25e-
2	V	length_2	m_1_length_2	հ	Random Parameter	Normal Variance 🔺	0.0000000	6.25e-
3	1	length_3	m_1_length_3	հ	Random Parameter	Normal Variance 🔺	-1.0000000	6.25e-
4	V	length_4	m_1_length_4	հ	Random Parameter	Normal Variance 🔺	0.4958683	6.25e-
5	V	length_5	m_1_length_5	հ	Random Parameter	Normal Variance 🔺	0.0347269	6.25e-
6		radius_1	m_1_radius_1	հ	Random Parameter	Normal Variance 🔺	0.0000000	1.0000
7		radius_2	m_1_radius_2	հ	Random Parameter	Normal Variance 🔺	0.0000000	0.1406
8		radius_3	m_1_radius_3	հ	Random Parameter	Normal Variance 🛕	0.0000000	0.1406
9	1	height	m_1_height	հ	Random Parameter	Normal Variance 🛕	1.0000000	6.25e-

- 8. Go to the **Select Output Responses** step.
- 9. Define Max_Disp, Volume and Max_Stress by selecting the options indicated in the image below from the **Evaluate From** column.

Even though Max_stress was not used in the Optimization, you will use it in the Stochastic study to check the reliability.

Active	Label	Varname	Expression	Evaluate From	Comment
1 🗸	Max_Disp	m_1_r_1	max(m_1_ds_1)	🧇 Fit, RBF (fit_4)	Data Source 1
2 🗸	Max_Stress	m_1_r_2	max(m_1_ds_2)	🧇 Fit, RBF (fit_4)	Data Source 2
3 🗸	Volume	m_1_r_3	m_1_ds_3[0]	🫸 Fit, RBF (fit_4)	Data Source 3

- 10. Go to the **Specifications** step.
- 11. In the work area, set the **Mode** to **Hammersley**.

Hammersley sampling belongs to the category of quasi-Monte Carlo methods. This method uses a quasi-random number generator, based on the Hammersley points, to uniformly sample a unit hypercube.

- 12. Click **Apply**.
- 13. Go to the **Evaluate** step.
- 14. Click *Evaluate Tasks* to execute all the runs and extract the results.
- 15. To study the method convergence:
 - a. Go back to the **Specifications** step and change the **Number of runs** to 300 and then 1000 in the **Settings** tab.
 - b. Go to the **Evaluate** step, and click *Evaluate Tasks*.
- 16. Go to the **Post-Processing** step.



Step 2: Post Processing the Results of the Stochastic Study

1. Click the **Integrity** tab to review the statistics of the input variables and the output responses around the nominal design.

Using the **Channel** selector, select the **Health** category to get a summarized view of statistics and spot eventual, missing, or bad values. The Range column can be useful to understand the spread of values in the data from the minimum to the maximum.

2. Click the *Distribution* tab to review a histogram of the Stochastic results.

Using the **Channel** selector, select **Length_1**.

The chart shows three pieces of information about the distribution of values for the selected input variable. The **Histogram** uses the left axis, and represents the frequency of runs yielding a sub-range of response values. The **Probability density** uses the right axis, and indicates the relative likelihood of the input variable to take a particular value. A higher value indicates that the values are more probable to occur. The **Cumulative distribution** is another curve that uses the right axis. It is equal to the integral of the Probability density. The value of the Cumulative distribution indicates what percentage of the data falls below the value's threshold. Note that the initial value of the CUF will always be 1.0. This is because all of the data will reside between the upper and lower bounds.



3. Review the Probability density and the Cumulative distribution of Max_Disp. Compare the distributions obtained with the different number of runs for Hammersley (100, 300, and 1000).

Note that the pattern of the distribution changes quite a bit from 100 to 300 runs, but very little from 300 to 1000 runs.





Hammersley, 100 runs



Hammersley, 300 runs





Hammersley, 1000 runs

4. Click the *Integrity* tab to review the statistics of the output responses around the nominal design.

The column range can be useful to understand the spread of values in the data from the minimum to the maximum.

	Label	Varname	Category	Points	Unique	No Values	Bad Values	Excluded	Range
1	<pre>[] ← length_1</pre>	m_1_length_1	Variable	1000	1000	0	0	0	0.1645263
2	Length_2	m_1_length_2	Variable	1000	1000	0	0	0	0.1495726
3	<pre>[] ← length_3</pre>	m_1_length_3	Variable	1000	1000	0	0	0	0.1577675
4	<pre>[] ← length_4</pre>	m_1_length_4	Variable	1000	1000	0	0	0	0.1590478
5	" length_5	m_1_length_5	Variable	1000	1000	0	0	0	0.1537191
6	∐ ← height	m_1_height	Variable	1000	1000	0	0	0	0.1480346
7	🔏 Max_Disp	m_1_r_1	Response	1000	1000	0	0	0	0.0413813
8	🔏 Max_Stress	m_1_r_2	Response	1000	1000	0	0	0	13.517775
9	🕼 Volume	m_1_r_3	Response	1000	1000	0	0	0	17740.778

- 5. Add a reliability.
 - a. Click the **Reliability** tab to estimate the Probability of failure for the output responses (probability for an output response to violate a user selected bound).
 - b. Click Add Reliability.
 - c. Define the reliability. The Bound Values are those from the Optimization problem definition (Max_Disp<=1.5).
 - Set **Response** to *Max_Disp(m_1_r_1)*.
 - Set **Bound Type** to **<=**.
 - For Bound Value, enter 1.50.

The results show the convergence of the Hammersley method in function of the number of runs. The convergence is enhanced by increasing the number of runs. As



the results obtained with 300 or 1000 runs are quite similar, you can assume that in this case 300 runs are enough to get accurate results.

	Active	Response		Bound Ty	pe	Bound Value	Reliability	Probability of Failure		
1	V	Max_Disp (m_1_r_1) 🔻	•	<= · ·	•	1.5000000	0.8100000	0.1900000		

Hammersley, 100 runs

	Active	Response	Bound	Туре	Bound Value	Reliability	Probability of Failure				
1	V	Max_Disp (m_1_r_1) 🔻	<=	•	1.5000000	0.7966667	0.2033333				
	L 200										

Hammersley, 300 runs

	Active	Response	Bound Type	Bound Value	Reliability	Probability of Failure
1	V	Max_Disp (m_1_r_1) 🛛 🔻	<= ▼	1.5000000	0.8090000	0.1910000
		1 1000				

Hammersley, 1000 runs

- d. Study the effects of bounds on the reliability by entering different values in the **Bound Value** columns.
- e. **Optional**: Check the reliability on Max_Stress.

	Active	Response	Bo	ound Ty	pe	Bound Value	Reliability	Probability of Failure
1	V	Max_Disp (m_1_r_1) 🛛 🔻		<= .	•	1.5000000	0.8090000	0.1910000
2	V	Max_Stress (m_1_r_2) 🔻		<= .	•	300.00000	1.0000000	0.0000000

6. Click the **Reliability Plot** tab to see the relationship between the desired threshold and the reliability of the system. The values reported in the **Reliability** column of the **Reliability** tab can be observed on these reliability curves. On the Max_Disp reliability curve, only 11.4% of the designs have a value above 1.5, which means 88.6% are below 1.5. A reliability of 95% can be reached by changing the threshold for Max_Disp to 1.5034.





Step 3: Reliability-Based Design Optimization Study (RBDO)

In this step, you will be searching for 95% reliability on the Optimization constraint (max_disp < 1.5 mm). You will use fitting functions as opposed to the exact solver to evaluate the output responses. Among the approximations, you will use the RBF that is created with MELS DOE. As a result, you will use the SORA method. You will continue using the six important input variables that all follow a normal distribution. You will start the Optimization from the previously obtained optimum solution with GRSM.

- 1. In the **Explorer**, right-click and select **Add** from the context menu.
- 2. In the Add HyperStudy dialog, select *Optimization* and click OK.
- 3. Go to the Select Input Variables step.
- 4. In the **Active** column, clear the *radius_1*, *radius_2* and *radius_3* check boxes.
- 5. In the **Nominal** column, copy the parameter values at the optimal design.
 - a. Go to the GRSM, 6 IV, Exact Solver Optimization, Evaluate step, Iteration History tab, and copy the length_1, length_2, length_3, length_4, and length_5 parameter values.
 - b. Go to the Select Input Variables step of the Optimization study you just created, and select the Nominal fields for length_1, length_2, length_3, length_4, and length_5. Right-click on the selected fields, and select Paste transpose fro the context menu.
 - c. For height, change the Nominal value to 1.0.
- 6. Click the *Distributions* tab, and set the **Distribution Role** to *Design with Random* for all active input variables.

	Active	Label	Varname	Distribution Role	Distribution	Α	В	Г
1	1	length_1	m_1_length_1	Design with Random	Normal Variance 🛕	-0.4764394	0.3906250	
2	V	length_2	m_1_length_2	Design with Random	Normal Variance 🔺	0.0000000	0.2500000	
3	V	length_3	m_1_length_3	Design with Random	Normal Variance 🔺	-1.0000000	0.2500000	
4	V	length_4	m_1_length_4	Design with Random	Normal Variance 🔺	0.4958683	0.2500000	
5	V	length_5	m_1_length_5	Design with Random	Normal Variance 🔺	0.0347269	0.2500000	
6		radius_1	m_1_radius_1	" [← Design	Normal Variance 🔺	0.0000000	1.0000000	
7		radius_2	m_1_radius_2	Ľ [← Design	Normal Variance 🔺	0.0000000	0.1406250	
8		radius_3	m_1_radius_3	∐ + Design	Normal Variance 🔺	0.0000000	0.1406250	
9	V	height	m_1_height	Design with Random	Normal Variance 🛕	1.0000000	0.2500000	

7. In the **B** column, enter 6.25e-04 for all active input variables.

	Active	Label	Varname	Distribution Role	Distribution	А	В
1	1	length_1	m_1_length_1	Design with Random	Normal Variance 🔺	-0.4764394	6.25e-04
2	v	length_2	m_1_length_2	Design with Random	Normal Variance 🔺	0.0000000	6.25e-04
3	1	length_3	m_1_length_3	Design with Random	Normal Variance 🔺	-1.0000000	6.25e-04
4	v	length_4	m_1_length_4	Design with Random	Normal Variance 🔺	0.4958683	6.25e-04
5	1	length_5	m_1_length_5	Design with Random	Normal Variance 🔺	0.0347269	6.25e-04
6		radius_1	m_1_radius_1	∐ + Design	Normal Variance 🔺	0.0000000	1.0000000
7		radius_2	m_1_radius_2	∐ + Design	Normal Variance 🔺	0.0000000	0.1406250
8		radius_3	m_1_radius_3	∐ + Design	Normal Variance 🔺	0.0000000	0.1406250
9	V	height	m_1_height	Design with Random	Normal Variance 🔺	1.0000000	6.25e-04

8. Go to the **Select Output Responses** tab.

- 9. Apply an objective on the Volume output response.
 - a. In the **Objectives** column for Volume, click **•**.
 - b. In the pop-up window, set **Type** to *Minimize* and click *OK*.
 - c. In the **Evaluate From** column for Volume, select *Fit, RBF (fit_4)*.

	Active	Label	Varname	Objectives	Constraints	Evaluate From	Expression
1		Max_Disp	m_1_r_1	•	•	> Solver	max(m_1_ds_1)
2		Max_Stress	m_1_r_2	•	0	> Solver	max(m_1_ds_2)
З		Volume	m_1_r_3	Minimize	0	🦘 Fit, RBF (fit_4)	m_1_ds_3[0]

- 10. Apply an objective on the Max_Disp output response.
 - a. In the **Constraints** column for Max_Disp, click •.
 - b. In the pop-up window, define the following and click **OK**.
 - Set **Type** to **Random**.
 - Set **Bound Type** to **<=**.
 - For Bound Value, enter 1.50.
 - For CFD Limit, enter 95.0.
 - c. In the **Evaluate From** column for Max_Disp, select *Fit, RBF (fit_4)*.

	Active	Label	Varname	Objectives	Constraints	Evaluate From
1	V	Max_Disp	m_1_r_1	•	<= 1.5000000	🫸 Fit, RBF (fit_4)
2	V	Max_Stress	m_1_r_2	•	0	> Solver
3	V	Volume	m_1_r_3	Minimize	•	🐤 Fit, RBF (fit_4)

- 11. In the Evaluate From column for Max_Stress, select Fit, RBF (fit_4).
- 12. Click Apply.
- 13. Go to the **Specifications** step.
- 14. In the work area, set the **Mode** to **Sequential Optimization and Reliability Assessment (SORA)**.



- 15. Click Apply.
- 16. Go to the **Evaluate** step.
- 17. Click *Evaluate Tasks*.
- 18. Click the *Iteration History* tab to review the results of the Optimization in a table.

A deterministic optimum is found first. As you can see in iteration 1, the displacement is at the constraint bound. This design is not 95% reliable as required in this probabilistic Optimization study. The SORA will work to make sure the constraint satisfies the probabilistic requirement. As seen in iteration 2, the design that meets 95% reliability is the one that has a max displacement shifted away from the bound of 1.5. Corresponding to this improvement, there is an increase in the objective value (volume).

	"]+ length_1	"]+ length_2	"]+ length_3	"]+ length_4	"]+ length_5	¶+ height	🕼 Max_Disp	🕼 Max_Stress	🕼 Volume	¥ Objective1	🛃 Constraint 1	🙀 Constraint 1 (V
1	0.1121318	0.1909208	-0.9868386	-0.0116877	0.2926825	0.9999957	1.4999989	232.20310	1666996.4	1666996.4	1.4999989	1.5127075
2	0.0789470	0.2024848	-1.0000000	0.0039868	0.3331738	1.0000000	1.4873662	229.00344	1672361.7	1672361.7	1.4873662	1.5000004
З	0.0796390	0.2025820	-1.0000000	0.0040144	0.3333208	1.0000000	1.4873652	229.00563	1672361.9	1672361.9	1.4873652	1.4999999

- 19. As SORA uses the approximation method for the reliability calculation, it is recommended that you make a validation Stochastic study after the Optimization.
 - a. In the **Explorer**, right-click on the Hammersley Stochastic study and select **Copy Approach** from the context menu.
 - b. In the Copy dialog, Label field, enter Validation SORA and click OK.
- 20. Copy and paste the optimum solution from the SORA Optimization into the **Nominal** column of the input variables for the Validation SORA Stochastic study.
 - a. Go to the SORA Optimization, **Evaluate** step, **Iteration History** tab, and copy the length_1, length_2, length_3, length_4, and length_5 parameter values.
 - b. Go to the Select Input Variables step of the Validation SORA Stochastic study you just created, and select the Nominal fields for length_1, length_2, length_3, length_4, and length_5. Right-click on the selected fields, and select Paste transpose fro the context menu.
- 21. Change the lower and upper bounds for every active input variable to match the dimensions' tolerances. For this tutorial, assume that the tolerances are within 0.05 mm.
 - a. For all active input variables, click in the **Nominal** field.
 - b. In the pop-up window, Value field, enter 0.05 and click +/-.
 - c. Click **OK** to accept the changes and close the pop-up window.
- 22. Go to the Select Output Responses step.
- 23. For Max_Stress, verify that **Evaluate From** is set to *Fit, RBF (fit_4)*.
- 24. Go to the **Specifications** step and verify that the **Mode** is set to **Hammersley** with 1000 runs.
- 25. Go to the **Evaluate** step, and click *Evaluate Tasks*.
- 26. Go to the **Post-Processing** step.
- 27. Add a reliability on Max_Disp.



- a. Click the *Reliability* tab.
- b. Click Add Reliability.
- c. Define the reliability.
 - Set **Response** to *Max_Disp*.
 - Set **Bound Type** to **<=**.
 - For Bound Value, enter 1.50.

The result confirms that the optimum solution found with SORA is reliable at 95%.

	Active	Response	Bound Type	Bound Value	Reliability	Probability of Failure
1	\checkmark	Max_Disp (m_1_r_1) 🔹	<= ▼	1.5000000	0.9490000	0.0510000

Last modified: v2017.2 (12.1156684)

